Comparison of Cauchy EDA and G3PCX Algorithms on the BBOB Noiseless Testbed

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ABSTRACT

Estimation-of-distribution algorithm equipped with Cauchy sampling distribution is compared with the generalized generation gap algorithm with parent centric crossover. Both algorithms were already presented at the 2009 black-box optimization benchmarking workshop where they often showed similar performance. This paper compares them in more detail and adds to the understanding of their key features and differences.

Categories and Subject Descriptors

G.1.6 [**Numerical Analysis**]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [**Analysis of Algorithms and Problem Complexity**]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization, Generalized generation gap, Parent-centric crossover, Estimation-of-distribution algorithm, Cauchy distribution

1. INTRODUCTION

Black-box optimization benchmarking (BBOB) workshop in 2009 set the stage for unified comparisons among black-box optimization algorithms. The 2010 issue of the BBOB methodology [4] added means for a detailed comparison of 2 algorithms. In this paper, two algorithms from the BBOB 2009 workshop are further compared. Data for both algorithms were taken from 2009 benchmarking, but the comparison is made using the new post-processing scripts and templates for BBOB 2010. Both algorithms fall into the class of evolutionary optimization algorithms and both use unimodal distribution to sample new candidate solutions. Yet there are many differences between them. The two algorithms chosen for the comparison are:

- The generalized generation gap (G3) model introduced by Deb in [1] and used with the parent centric crossover operator (PCX) introduced in [2]. The performance of the G3PCX algorithm on the BBOB 2009 noiseless test suite was reported in [8].
- The estimation-of-distribution algorithm (EDA) with Cauchy sampling distribution (Cauchy EDA) [7].

Both algorithms can be described as local search techniques[1]. In the final algorithm presentation at the BBOB 2009 workshop, neither of the two algorithms belonged to the top-notch algorithms; nevertheless their results were often very similar despite the differences between the two algorithms. This paper emphasizes the differences between them and their impact on the search perfomance.

In the next section, both algorithms are shortly described and their differences are emphasized. Sec. 3 contains all the results used to compare the algorithms and their discussion. After the presentation of the time demands of both algorithms in Sec. 4, Sec. 5 concludes the paper.

2. ALGORITHM PRESENTATION

The exact descriptions of the algorithms along with the parameter settings can be found in [8] and [7], respectively. The algorithms differ foremost in the following aspects:

- The sampling distribution in the used variant of G3PCX is Gaussian and centered around the best solution found so far, while the EDA uses Cauchy distribution centered around the mean of selected parents.
- G3PCX is a steady-state model, with small incremental changes made to the population each generation. Cauchy EDA uses generational scheme where the whole population is replaced by the offspring each generation.
- While Cauchy EDA uses relatively large set of selected individuals to estimate the shape of the distribution, G3PCX uses only 3 parents to compute the distribution parameters.

[1]Note that the local-search behaviour is not generally a feature of the G3PCX algorithm. Only the particular implementation of this algorithm used in this comparison is such local-search heavy.

GECCO'10, July 7–11, 2010, Portland, Oregon, USA.

To fight the premature convergence, Cauchy EDA uses a constant multiplier to enlarge the variance of the distribution (as suggested in [6]). In case of G3PCX no such remedies are used. For both algorithms, the crafting effort CrE= 0.

3. RESULTS

Results from experiments according to [4] on the benchmark functions given in [3, 5] are presented in Figures 1, 2 and 3 and in Table 1. The **expected running time (ERT)**, used in the figures and table, depends on a given target function value, $f_{\mathrm{t}} = f_{\mathrm{opt}} + \Delta f$, and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach f_{t}, summed over all trials and divided by the number of trials that actually reached f_{t} [4, 9]. **Statistical significance** is tested with the rank-sum test for a given target Δf_{t} (10^{-8} in Figure 1) using, for each trial, either the number of needed function evaluations to reach Δf_{t} (inverted and multiplied by -1), or, if the target was not reached, the best Δf-value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

G3PCX ouperforms Cauchy EDA on functions 1, 5, 6, 8, 9, 12, 16, 21, 22, 23, while Cauchy EDA beats G3PCX on functions 2, 7, 10, 13, 17, 18, i.e. in this small competition the G3PCX wins 10:6. (The results on the other functions are mixed, or neither algorithm solved the problem successfully.)

In Fig. 1, we can often see a peak at the beginning of the ERT ratio lines (funtions 1, 5, 7, 11, 13, 14). The peak means that Cauchy EDA is much less efficient in the beginning of the search than the G3PCX algorithm. This is probably due to the fact that the probabilistic model used by Cauchy EDA needs some time to adapt to the fitness landscape, so that while G3PCX improves the best-so-far solution rather quickly right from the beginning, Cauchy EDA blunders. After finding the right model, the Cauchy EDA is sometimes able to close the gap and take the lead (e.g. for both ellipsoid functions 2 and 10 and for discus function 11 for dimensions lower than 20, or for the sharp ridge function 13).

G3PCX failed on functions 7 (Step-ellipsoid) and on both Schaffer's functions 17 and 18. It seems that for these functions the global point of view represented by a unimodal Cauchy distribution is a better approach. Also for function 13 (sharp ridge problem), the global probabilistic model seems better, at least for tight target levels.

Interesting results may be found for function 14. G3PCX algorithm is orders of magnitude faster than Cauchy EDA for a broad range of target levels. But for target levels at about 10^{-5} and tighter, Cauchy EDA takes over and its results are much better. It seems that G3PCX is not even able to find some of the tighter target levels. One explanation for that might be that the stopping criterion for G3PCX is not set properly and actually prevents the algorithm from finding these target levels. Another explanation may be that for these tight target levels the population of G3PCX often converges to certain subspace of the search space and the algorithm stagnates.

Another note can be made on the variance enlargment constant used by Cauchy EDA. It was set to be approximately optimal for the Rosenbrock's function. However, such setting may be too large for other functions. The ERT ratio for sphere function shows that with increasing problem dimensionality the gap between the algorithms gets larger. Also the results for function 21 and 22 (and possibly for functions 16 and 23) suggest, that the slow convergence of Cauchy EDA prevents it to be restarted more often which is the key to solve these problems; on the contrary, G3PCX converges probably much faster and is thus restarted more often which gives it a chance to have higher success rate.

Looking at Fig. 3, it can be stated that G3PCX beats CauchyEDA mainly on the separable functions (where for 20D, we can expect the G3PCX to be faster than Cauchy EDA at least 80 % of time regardless of the target level), on moderate functions (where G3PCX would be winner about 75 % of time), and on weak-structure functions (where Cauchy EDA almost does not work at all). On the other hand, Cauchy EDA has higher success rates on ill-conditioned and multi-modal functions, but compared to G3PCX and other algorithms it is orders of magnitude slower.

Table 2: The average time demands per function evaluation (in microseconds) of the two compared algorithms.

Dim	2	3	5	10	20	40
G3PCX	470	443	420	411	540	750
CauchyEDA	51	17	9	9	11	NA

4. CPU TIMING EXPERIMENTS

The time requirements of both algorithms are taken from the respective articles, [8] and [7]. The multistart algorithm was run with the maximal number of evaluations set to 10^5, the basic algorithm was restarted for at least 30 seconds. The experiment was conducted on Intel Core 2 CPU, T5600, 1.83 GHz, 1 GB RAM with Windows XP SP3 in MATLAB R2007b. The comparison of the average time demands per function evaluation are shown in Table 2.

The differences in the average time needed for function evaluation are probably caused by the different offspring population sizes. While G3PCX generates 2 offspring each generation, Cauchy EDA generates tens or hundreds. This means that the evaluation routine is called less often in Cauchy EDA than in G3PCX and can take advantage of the MATLAB matrix processing capabilities to a larger extent.

5. CONCLUSIONS

Two restarted local-search algorithms, G3PCX and Cauchy EDA, were compared using the BBOB 2010 methodology. Neither of the algorithms is successful in solving all the benchmark functions. The results are mixed, but the G3PCX gives better results for a larger subset of the benchmark functions. This is probably caused by the fact that it converges faster and can be restarted more often, when needed.

Acknowledgements

The author is supported by the Grant Agency of the Czech Republic with the grant no. 102/08/P094 entitled "Machine learning methods for solution construction in evolutionary algorithms".

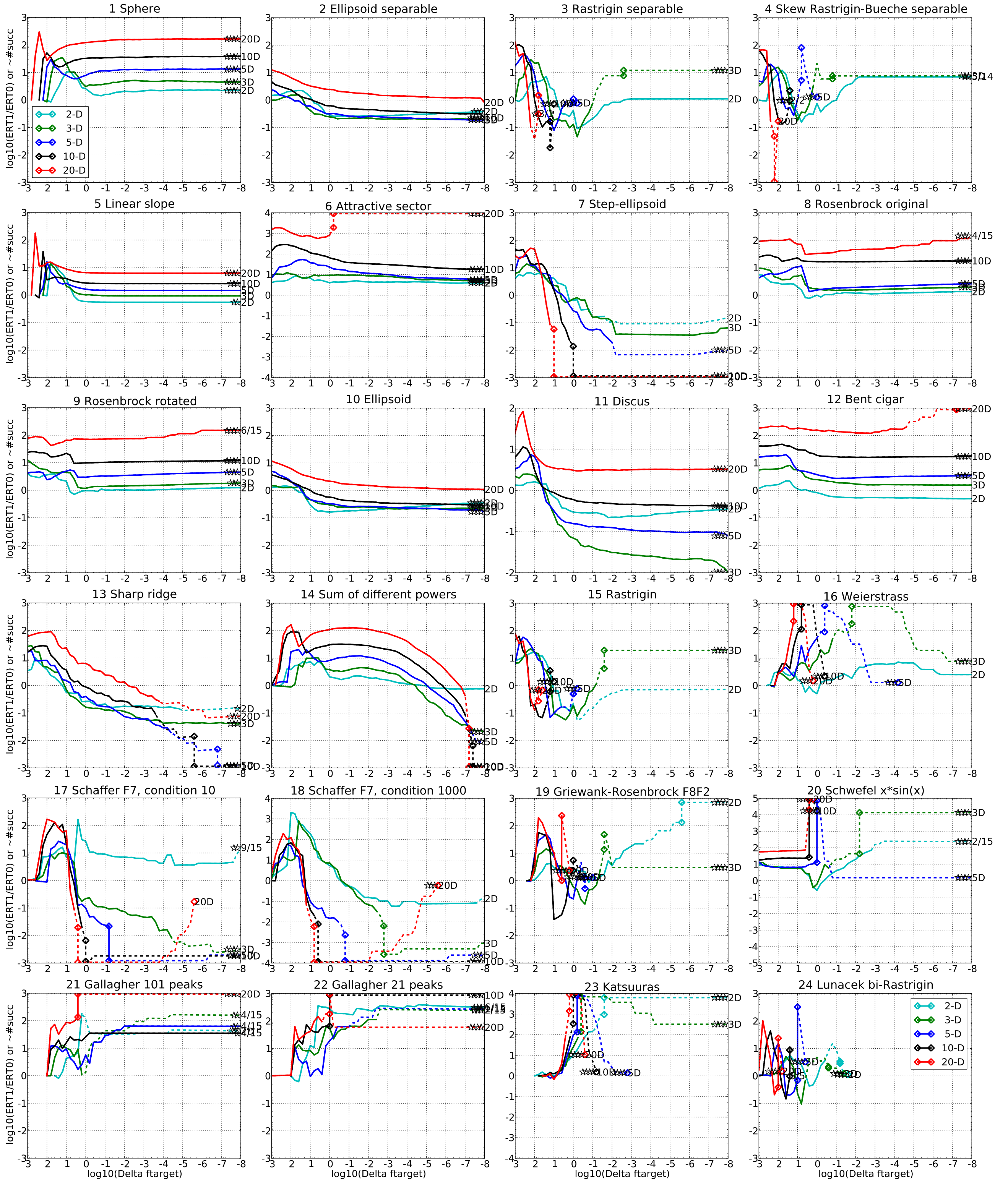

Figure 1: ERT ratio of CauchyEDA divided by G3PCX versus $\log_{10}(\Delta f)$ for f_1–f_{24} in 2, 3, 5, 10, 20, 40-D. Ratios $< 10^0$ indicate an advantage of CauchyEDA, smaller values are always better. The line gets dashed when for any algorithm the ERT exceeds thrice the median of the trial-wise overall number of f-evaluations for the same algorithm on this function. Symbols indicate the best achieved Δf-value of one algorithm (ERT gets undefined to the right). The dashed line continues as the fraction of successful trials of the other algorithm, where 0 means 0% and the y-axis limits mean 100%, values below zero for CauchyEDA. The line ends when no algorithm reaches Δf anymore. The number of successful trials is given, only if it was in $\{1 \ldots 9\}$ for CauchyEDA (1st number) and non-zero for G3PCX (2nd number). Results are significant with $p = 0.05$ for one star and $p = 10^{-\#\star}$ otherwise, with Bonferroni correction within each figure.

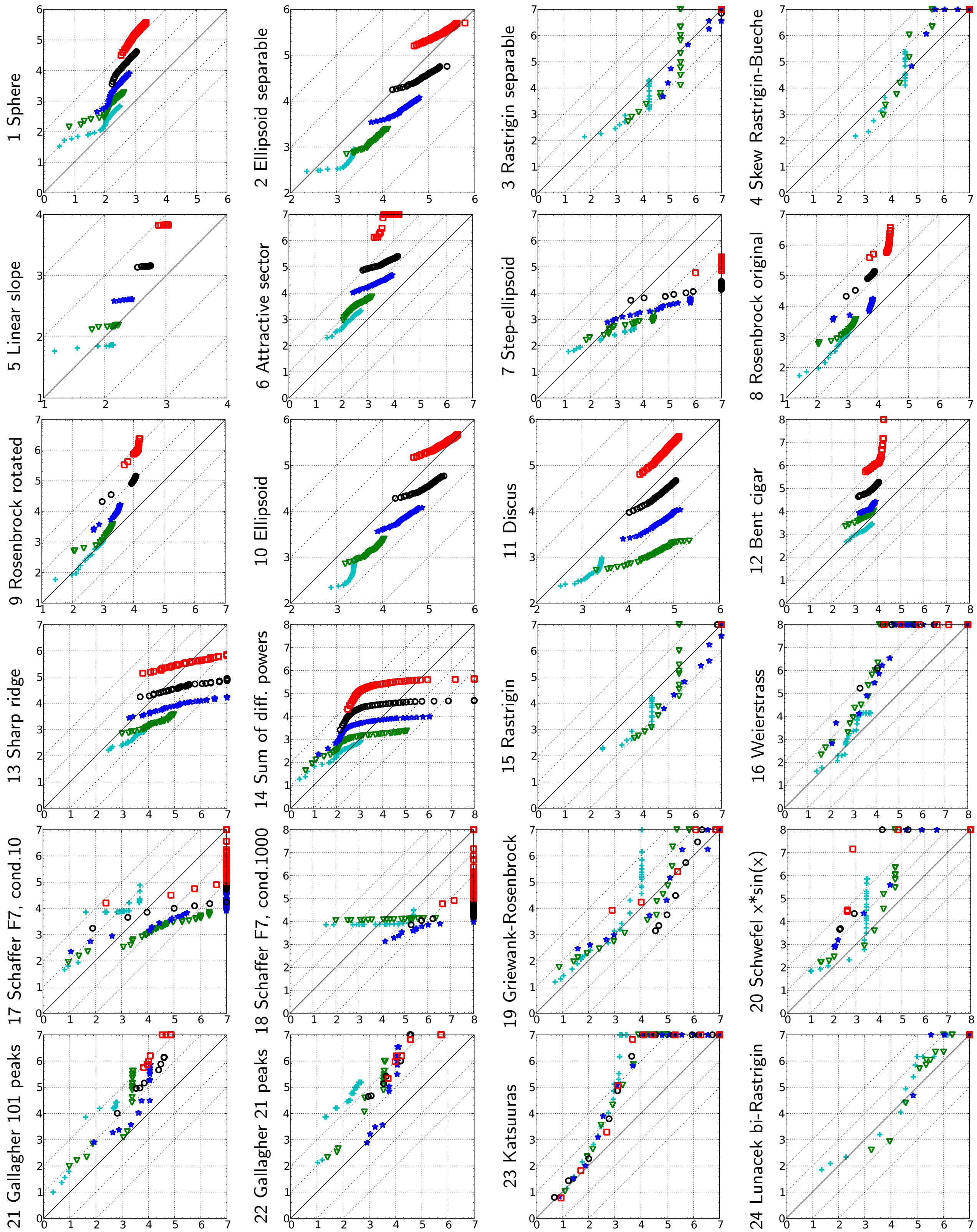

Figure 2: Expected running time (ERT in log10 of number of function evaluations) of CauchyEDA versus G3PCX for 46 target values $\Delta f \in [10^{-8}, 10]$ in each dimension for functions f_1–f_{24}. Markers on the upper or right egde indicate that the target value was never reached by CauchyEDA or G3PCX respectively. Markers represent dimension: 2:+, 3:▽, 5:⋆, 10:○, 20:□, 40:◇.

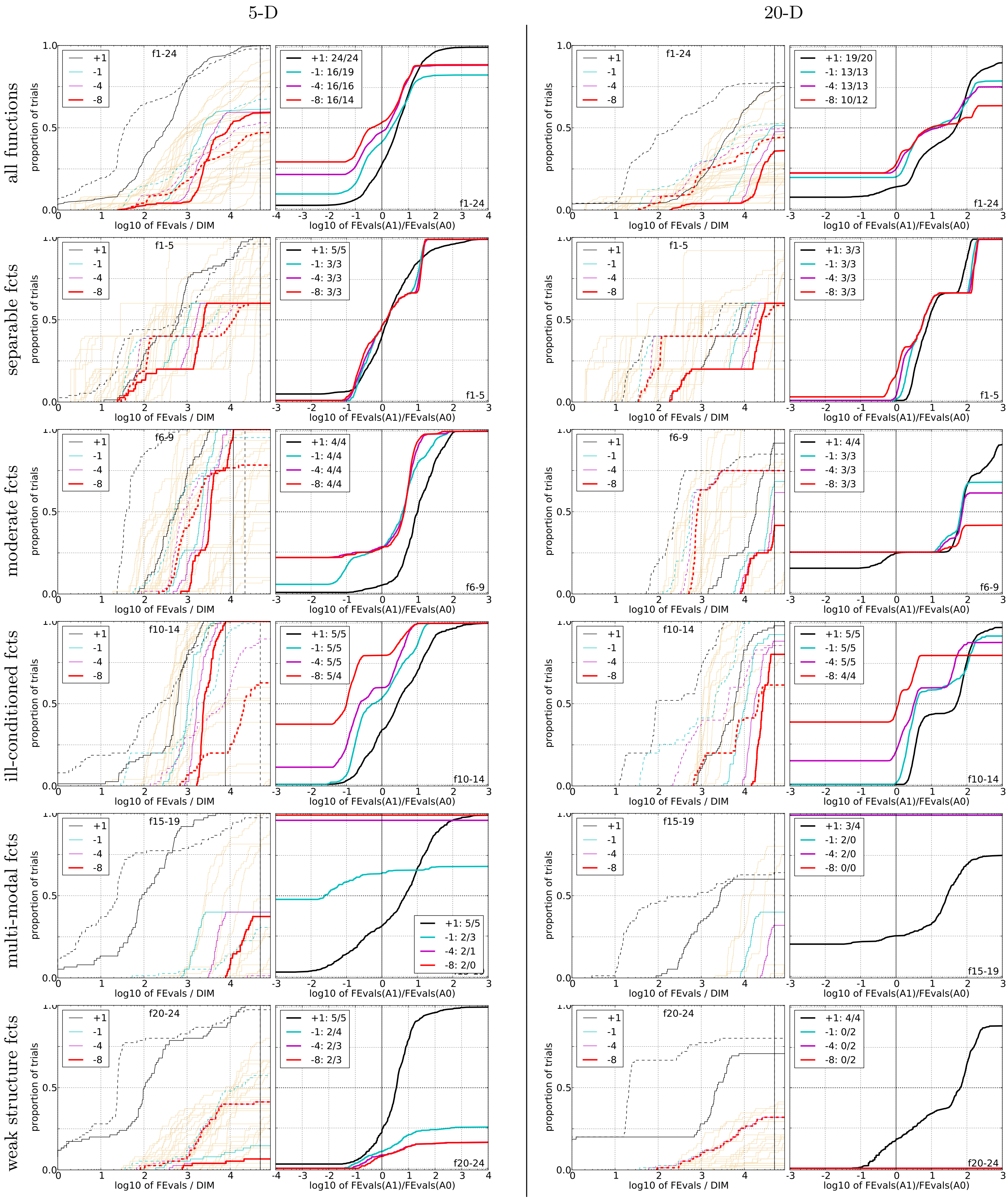

Figure 3: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right). Left sub-columns: ECDF of the number of function evaluations divided by dimension D (FEvals/D) to reach a target value $f_{\rm opt} + \Delta f$ with $\Delta f = 10^k$, where $k \in \{1, -1, -4, -8\}$ is given by the first value in the legend, for CauchyEDA (solid) and G3PCX (dashed). Light beige lines show the ECDF of FEvals for target value $\Delta f = 10^{-8}$ of algorithms benchmarked during BBOB-2009. Right sub-columns: ECDF of FEval ratios of CauchyEDA divided by G3PCX, all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being > 0 or < 1. The legends indicate the number of functions that were solved in at least one trial (CauchyEDA first).

5-D

Δf	10^{1}	10^{0}	10^{-1}	10^{-3}	10^{-5}	10^{-7}	#succ
f_1	11	12	12	12	12	12	15/15
0: G3P	5.2	$12^{\star 3}$	$15^{\star 3}$	$25^{\star 3}$	$35^{\star 3}$	$45^{\star 3}$	15/15
1: Cau	$41^{\star 3}$	$90^{\star 3}$	$170^{\star 3}$	$310^{\star 3}$	$460^{\star 3}$	$600^{\star 3}$	15/15
f_2	83	87	88	90	92	94	15/15
0: G3P	$69^{\star 3}$	$150^{\star 3}$	$220^{\star 3}$	$340^{\star 3}$	$470^{\star 3}$	$620^{\star 3}$	15/15
1: Cau	$42^{\star 3}$	$49^{\star 3}$	$58^{\star 3}$	$80^{\star 3}$	$100^{\star 3}$	$120^{\star 3}$	15/15
f_3	720	1600	1600	1600	1700	1700	15/15
0: G3P	$84^{\star 2}$	$2100^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
1: Cau	$6.7^{\star 2}$	$2200^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
f_4	810	1600	1700	1800	1900	1900	15/15
0: G3P	$76^{\star 2}$	$2200^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
1: Cau	$85^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
f_5	10	10	10	10	10	10	15/15
0: G3P	$14^{\star 3}$	$25^{\star 3}$	$27^{\star 3}$	$28^{\star 3}$	$28^{\star 3}$	$28^{\star 3}$	15/15
1: Cau	$39^{\star 3}$	$41^{\star 3}$	$41^{\star 3}$	$41^{\star 3}$	$41^{\star 3}$	$41^{\star 3}$	15/15
f_6	110	210	280	580	1000	1300	15/15
0: G3P	$2.4^{\star 2}$	$3^{\star 2}$	$4.8^{\star 2}$	$4.7^{\star 2}$	$5.1^{\star 2}$	$5.5^{\star 2}$	15/15
1: Cau	$92^{\star 3}$	$69^{\star 3}$	$68^{\star 3}$	$47^{\star 3}$	$35^{\star 3}$	$34^{\star 3}$	15/15
f_7	24	320	1200	1600	1600	1600	15/15
0: G3P	$19^{\star 3}$	18	$45^{\star 3}$	$410^{\star 3}$	$410^{\star 3}$	$410^{\star 3}$	2/15
1: Cau	$33^{\star 3}$	$4.9^{\star 3}$	$2.4^{\star 2}$	$2.9^{\star 3}$	$2.9^{\star 3}$	$3.4^{\star 3}$	15/15
f_8	73	270	340	390	410	420	15/15
0: G3P	$4.6^{\star 2}$	$20^{\star 3}$	$18^{\star 3}$	$17^{\star 3}$	$16^{\star 3}$	$16^{\star 3}$	15/15
1: Cau	$49^{\star 3}$	$31^{\star 3}$	$33^{\star 3}$	$34^{\star 3}$	$37^{\star 3}$	$40^{\star 3}$	15/15
f_9	35	130	210	300	340	370	15/15
0: G3P	$14^{\star 3}$	$18^{\star 3}$	$14^{\star 3}$	$11^{\star 3}$	$10^{\star 3}$	$9.8^{\star 3}$	15/15
1: Cau	$71^{\star 3}$	$54^{\star 3}$	$45^{\star 3}$	$41^{\star 3}$	$42^{\star 3}$	$43^{\star 3}$	15/15
f_{10}	350	500	570	630	830	880	15/15
0: G3P	$22^{\star 3}$	$27^{\star 3}$	$36^{\star 3}$	$49^{\star 3}$	$51^{\star 3}$	$64^{\star 3}$	15/15
1: Cau	$11^{\star 3}$	$9^{\star 3}$	$9.4^{\star 3}$	$12^{\star 3}$	$11^{\star 3}$	$13^{\star 3}$	15/15
f_{11}	140	200	760	1200	1500	1700	15/15
0: G3P	$55^{\star 3}$	$110^{\star 3}$	$45^{\star 3}$	$49^{\star 3}$	$56^{\star 3}$	$61^{\star 3}$	14/15
1: Cau	$18^{\star 3}$	$17^{\star 3}$	$6^{\star 3}$	$5.3^{\star 3}$	$5.6^{\star 3}$	$5.9^{\star 3}$	15/15
f_{12}	110	270	370	460	1300	1500	15/15
0: G3P	$14^{\star 3}$	$11^{\star 2}$	$13^{\star 3}$	$12^{\star 3}$	$5.2^{\star 3}$	$5.1^{\star 3}$	15/15
1: Cau	$79^{\star 3}$	$41^{\star 3}$	$35^{\star 3}$	$38^{\star 3}$	$17^{\star 3}$	$17^{\star 3}$	15/15
f_{13}	130	190	250	1300	1800	2300	15/15
0: G3P	$14^{\star 3}$	$60^{\star 3}$	$150^{\star 3}$	$120^{\star 3}$	$590^{\star 3}$	$\infty^{\star 3}$	0/15
1: Cau	$21^{\star 3}$	$24^{\star 3}$	$25^{\star 3}$	$7.4^{\star 3}$	$7.3^{\star 3}$	$7.3^{\star 3}$	15/15
f_{14}	9.8	41	58	140	250	480	15/15
0: G3P	1.7	$3.5^{\star 3}$	$3.5^{\star 3}$	$5^{\star 3}$	$26^{\star 3}$	$390^{\star 3}$	3/15
1: Cau	23	$29^{\star 3}$	$40^{\star 3}$	$33^{\star 3}$	$28^{\star 3}$	$19^{\star 3}$	15/15
f_{15}	510	9300	1.9e4	2.0e4	2.1e4	2.1e4	14/15
0: G3P	$130^{\star 2}$	$370^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
1: Cau	$12^{\star 3}$	$190^{\star 2}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
f_{16}	120	610	2700	1.0e4	1.2e4	1.2e4	15/15
0: G3P	1	$22^{\star}$	44	$350^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
1: Cau	5.6	$1200^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
f_{17}	5.2	210	900	3700	6400	7900	15/15
0: G3P	2.2	130	$290^{\star 2}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
1: Cau	44	$13^{\star 3}$	$7^{\star 3}$	$4.3^{\star 3}$	$5.3^{\star 3}$	$13^{\star 3}$	14/15
f_{18}	100	380	4000	9300	1.1e4	1.2e4	15/15
0: G3P	130	$800^{\star 2}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
1: Cau	$13^{\star 3}$	$12^{\star 3}$	$2.4^{\star}$	$2.7^{\star 3}$	$3.7^{\star 3}$	$8.6^{\star 3}$	14/15
f_{19}	1	1	240	1.2e5	1.2e5	1.2e5	15/15
0: G3P	39	9.5e4	$1.4e4^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
1: Cau	300	2.1e4	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
f_{20}	16	850	3.8e4	5.4e4	5.5e4	5.5e4	14/15
0: G3P	$7.4^{\star 2}$	$36^{\star 2}$	$88^{\star 3}$	$62^{\star 3}$	$61^{\star 3}$	$61^{\star 3}$	1/15
1: Cau	$48^{\star 3}$	$460^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
f_{21}	41	1200	1700	1700	1700	1800	14/15
0: G3P	2.1	4.7	6.8	6.7	6.7	6.6	15/15
1: Cau	20	27	$190^{\star}$	$420^{\star 2}$	$420^{\star 2}$	$410^{\star 2}$	4/15
f_{22}	71	390	940	1000	1000	1100	14/15
0: G3P	12	15	13	$12^{\star}$	$12^{\star 2}$	$12^{\star 2}$	15/15
1: Cau	11	280	$780^{\star 2}$	$3500^{\star 2}$	$3400^{\star 2}$	$3300^{\star 2}$	1/15
f_{23}	3	520	1.4e4	3.2e4	3.3e4	3.4e4	15/15
0: G3P	2.6	2.4	$8.6^{\star}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
1: Cau	2.2	$230^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
f_{24}	1600	2.2e5	6.4e6	9.6e6	1.3e7	1.3e7	3/15
0: G3P	$44^{\star 2}$	∞	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
1: Cau	$30^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15

20-D

Δf	10^{1}	10^{0}	10^{-1}	10^{-3}	10^{-5}	10^{-7}	#succ
f_1	43	43	43	43	43	43	15/15
0: G3P	$8^{\star 3}$	$13^{\star 3}$	$18^{\star 3}$	$27^{\star 3}$	$37^{\star 3}$	$48^{\star 3}$	15/15
1: Cau	$730^{\star 3}$	$1600^{\star 3}$	$2500^{\star 3}$	$4300^{\star 3}$	$6100^{\star 3}$	$7800^{\star 3}$	15/15
f_2	380	390	390	390	390	390	15/15
0: G3P	$130^{\star 3}$	$210^{\star 3}$	$320^{\star 3}$	$550^{\star 3}$	$760^{\star 3}$	$990^{\star 3}$	14/15
1: Cau	$410^{\star 3}$	$510^{\star 3}$	$610^{\star 3}$	$800^{\star 3}$	$990^{\star 3}$	$1200^{\star 3}$	15/15
f_3	5100	7600	7600	7600	7600	7700	15/15
0: G3P	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
1: Cau	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
f_4	4700	7600	7700	7700	7800	1.4e5	9/15
0: G3P	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
1: Cau	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
f_5	41	41	41	41	41	41	15/15
0: G3P	$19^{\star 3}$	$25^{\star 3}$	$26^{\star 3}$	$27^{\star 3}$	$27^{\star 3}$	$27^{\star 3}$	15/15
1: Cau	$160^{\star 3}$	$170^{\star 3}$	$170^{\star 3}$	$170^{\star 3}$	$170^{\star 3}$	$170^{\star 3}$	15/15
f_6	1300	2300	3400	5200	6700	8400	15/15
0: G3P	1.4	1.5	1.5	$1.5^{\star 2}$	$1.7^{\star 3}$	$1.7^{\star 3}$	15/15
1: Cau	$1000^{\star 3}$	$1300^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
f_7	1400	4300	9500	1.7e4	1.7e4	1.7e4	15/15
0: G3P	$760^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
1: Cau	$44^{\star 3}$	$29^{\star 3}$	$18^{\star 3}$	$14^{\star 3}$	$14^{\star 3}$	$14^{\star 3}$	15/15
f_8	2000	3900	4000	4200	4400	4500	15/15
0: G3P	$2.6^{\star 3}$	$5.4^{\star 3}$	$5.5^{\star 3}$	$5.5^{\star 3}$	$5.5^{\star 3}$	$5.6^{\star 3}$	15/15
1: Cau	$190^{\star 3}$	$180^{\star 3}$	$210^{\star 3}$	$260^{\star 3}$	$360^{\star 3}$	$540^{\star 3}$	4/15
f_9	1700	3100	3300	3500	3600	3700	15/15
0: G3P	$2.9^{\star 2}$	$3.8^{\star 3}$	$4^{\star 3}$	$4.1^{\star 3}$	$4.1^{\star 3}$	$4.2^{\star 3}$	15/15
1: Cau	$190^{\star 3}$	$270^{\star 3}$	$290^{\star 3}$	$310^{\star 3}$	$470^{\star 3}$	$630^{\star 3}$	6/15
f_{10}	7400	8700	1.1e4	1.5e4	1.7e4	1.7e4	15/15
0: G3P	$6.5^{\star 3}$	$10^{\star 3}$	$12^{\star 3}$	$15^{\star 3}$	$18^{\star 3}$	$23^{\star 3}$	15/15
1: Cau	$20^{\star 3}$	$22^{\star 3}$	$20^{\star 3}$	$20^{\star 3}$	$21^{\star 3}$	$25^{\star 3}$	15/15
f_{11}	1000	2200	6300	9800	1.2e4	1.5e4	15/15
0: G3P	$18^{\star 3}$	$14^{\star 3}$	$7^{\star 3}$	$7.1^{\star 3}$	$7.6^{\star 3}$	$8^{\star 3}$	15/15
1: Cau	$64^{\star 3}$	$44^{\star 3}$	$22^{\star 3}$	$22^{\star 3}$	$25^{\star 3}$	$26^{\star 3}$	15/15
f_{12}	1000	1900	2700	4100	1.2e4	1.4e4	15/15
0: G3P	2.7	2.8	3	$2.9^{\star 2}$	1.2	1.3	15/15
1: Cau	$510^{\star 3}$	$440^{\star 3}$	$420^{\star 3}$	$380^{\star 3}$	$390^{\star 3}$	$1100^{\star 3}$	0/15
f_{13}	650	2000	2800	1.9e4	2.4e4	3.0e4	15/15
0: G3P	$9.3^{\star}$	$17^{\star 2}$	$43^{\star 3}$	$47^{\star 3}$	$130^{\star 3}$	$330^{\star 3}$	1/15
1: Cau	$210^{\star 3}$	$100^{\star 3}$	$100^{\star 3}$	$23^{\star 3}$	$23^{\star 3}$	$23^{\star 3}$	15/15
f_{14}	75	240	300	930	1600	1.6e4	15/15
0: G3P	$4.1^{\star 3}$	$2.4^{\star 3}$	$2.8^{\star 3}$	$2.7^{\star 3}$	$13^{\star 3}$	$59^{\star 3}$	0/15
1: Cau	$280^{\star 3}$	$270^{\star 3}$	$350^{\star 3}$	$210^{\star 3}$	$180^{\star 3}$	$25^{\star 3}$	15/15
f_{15}	3.0e4	1.5e5	3.1e5	3.2e5	4.5e5	4.6e5	15/15
0: G3P	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
1: Cau	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
f_{16}	1400	2.7e4	7.7e4	1.9e5	2.0e5	2.2e5	15/15
0: G3P	17	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
1: Cau	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
f_{17}	63	1000	4000	3.1e4	5.6e4	8.0e4	15/15
0: G3P	$4^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
1: Cau	$260^{\star 3}$	$120^{\star 3}$	$62^{\star 3}$	$16^{\star 3}$	$23^{\star 3}$	$\infty^{\star 3}$	0/15
f_{18}	620	4000	2.0e4	6.8e4	1.3e5	1.5e5	15/15
0: G3P	$7100^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
1: Cau	$96^{\star 3}$	$42^{\star 3}$	$15^{\star 3}$	$12^{\star 3}$	$38^{\star 3}$	$\infty^{\star 3}$	0/15
f_{19}	1	1	3.4e5	6.2e6	6.7e6	6.7e6	15/15
0: G3P	800	∞	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
1: Cau	8400	∞	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
f_{20}	82	4.6e4	3.1e6	5.5e6	5.6e6	5.6e6	14/15
0: G3P	$5^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
1: Cau	$340^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
f_{21}	560	6500	1.4e4	1.5e4	1.6e4	1.8e4	15/15
0: G3P	12	7.2	5.1	4.9	4.6	4.1	15/15
1: Cau	$1000^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
f_{22}	470	5600	2.3e4	2.5e4	2.7e4	1.3e5	12/15
0: G3P	$11^{\star}$	6.6	23	22	20	4	9/15
1: Cau	$470^{\star 3}$	$1200^{\star 2}$	$\infty^{\star}$	$\infty^{\star}$	$\infty^{\star}$	$\infty^{\star 3}$	0/15
f_{23}	3.2	1600	6.7e4	4.9e5	8.1e5	8.4e5	15/15
0: G3P	2.8	$7.8^{\star 2}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
1: Cau	1.9	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
f_{24}	1.3e6	7.5e6	5.2e7	5.2e7	5.2e7	5.2e7	3/15
0: G3P	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15
1: Cau	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	$\infty^{\star 3}$	0/15

Table 1: Expected running time (ERT in number of function evaluations) divided by the best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values for functions f_1–f_{24}. The median number of conducted function evaluations is additionally given in *italics*, if $\mathrm{ERT}(10^{-7}) = \infty$. #succ is the number of trials that reached the final target $f_{\mathrm{opt}} + 10^{-8}$. 0: G3P is G3PCX and 1: Cau is CauchyEDA. Bold entries are statistically significantly better compared to the other algorithm, with $p = 0.05$ or $p = 10^{-k}$ where $k > 1$ is the number following the $\star$ symbol, with Bonferroni correction of 48.

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